19.61. Model: The closed cycle in this heat engine includes adiabatic process $1 \rightarrow 2$, isobaric process $2 \rightarrow 3$, and isochoric process $3 \rightarrow 1$. For a diatomic gas, $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$, and $\gamma = \frac{7}{5} = 1.4$.

Visualize: Please refer to Figure P19.61.

Solve: **(a)** We can find the temperature T_2 from the ideal-gas equation as follows:

$$T_2 = \frac{p_2 V_2}{nR} = \frac{(4.0\times10^5 \text{ Pa})(1.0\times10^{-3} \text{ m}^3)}{(0.020 \text{ mol})(8.31 \text{ J / mol K})} = 2407 \text{ K}$$

We can use the equation $p_2 V_2^\gamma = p_1 V_1^\gamma$ to find V_1,

$$V_1 = V_2\left(\frac{p_2}{p_1}\right)^{1/\gamma} = (1.0\times10^{-3} \text{ m}^3)\left(\frac{4.0\times10^5 \text{ Pa}}{1.0\times10^5 \text{ Pa}}\right)^{1/1.4} = 2.692\times10^{-3} \text{ m}^3$$

The ideal-gas equation can now be used to find T_1,

$$T_1 = \frac{p_1 V_1}{nR} = \frac{(1.0\times10^5 \text{ Pa})(2.692\times10^{-3} \text{ m}^3)}{(0.020 \text{ mol})(8.31 \text{ J / mol K})} = 1620 \text{ K}$$

At point 3, $V_3 = V_1$ so we have

$$T_3 = \frac{p_3 V_3}{nR} = \frac{(4\times10^5 \text{ Pa})(2.692\times10^{-3} \text{ m}^3)}{(0.020 \text{ mol})(8.31 \text{ J / mol K})} = 6479 \text{ K}$$

(b) For adiabatic process $1 \rightarrow 2$, $Q = 0$ J, $\Delta E_{th} = -W_s$, and

$$W_s = \frac{p_2 V_2 - p_1 V_1}{1-\gamma} = \frac{nR(T_2 - T_1)}{1-\gamma} = \frac{(0.020 \text{ mol})(8.31 \text{ J / mol K})(2407 \text{ K} - 1620 \text{ K})}{(1-1.4)} = -327.0 \text{ J}$$

For isobaric process $2 \rightarrow 3$,

$$Q = nC_P\Delta T = n\left(\tfrac{7}{2}R\right)(\Delta T) = (0.020 \text{ mol})\tfrac{7}{2}(8.31 \text{ J / mol K})(6479 \text{ K} - 2407 \text{ K}) = 2369 \text{ J}$$

$$\Delta E_{th} = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)\Delta T = 1692 \text{ J}$$

The work done is the area under the p-versus-V graph. Hence,

$$W_s = (4.0\times10^5 \text{ Pa})(2.692\times10^{-3} \text{ m}^3 - 1.0\times10^{-3} \text{ m}^3) = 677 \text{ J}$$

For isochoric process $3 \rightarrow 1$, $W_s = 0$ J and

$$\Delta E_{th} = Q = nC_V\Delta T = (0.020 \text{ mol})\left(\tfrac{5}{2}\right)(8.31 \text{ J / mol K})(1620 \text{ K} - 6479 \text{ K}) = -2019 \text{ J}$$

	ΔE_{th} (J)	W_s (J)	Q (J)
$1 \rightarrow 2$	327	–327	0
$2 \rightarrow 3$	1692	677	2369
$3 \rightarrow 1$	–2019	0	–2019
Net	0	350	350

(c) The engine's thermal efficiency is

$$\eta = \frac{350 \text{ J}}{2369 \text{ J}} = 0.148 = 14.8\%$$